# Mathematics Calculation and 

## Number Fluency Policy

## Curriculum MMXIV

## Chacewater School



## Introduction

The purpose of this document is to build on the successes of the Calculation Policy which was devised by the Falmouth Learning Network in 2008 and create a personalised, updated policy reflecting the requirements of the new curriculum and more importantly, the needs of our pupils.

This policy aims to develop, model and explain core understandings and mathematical principles and progression to ensure consistency in the teaching and learning of mathematics in our schools.

The focus of this policy is the calculation of the four mathematical operations with an emphasis on written strategies to clarify processes and understanding and to make direct links to mental calculating. It is crucial that these mental strategies are discretely taught and linked to written strategies and not confined to starter activities in lessons.

The overall aims of this policy are that, when children leave primary school they:

- have a secure knowledge of number facts and a good understanding of the four operations supported by a fluency and understanding of the fundamentals of mathematics
- includes the idea of variation theory which is linked to mastery and has been adapted following the joint England Shanghai maths project of 2014/15 which the school was a part of. This is taking known facts and repeating these to support the application of skills.
- know the best strategy to use, estimate before calculating, systematically break problems down into a series of simpler steps with perseverance and use estimation and rounding to check that an answer is reasonable
- are able to use this knowledge and understanding to carry out calculations mentally, solve problems of increasing complexity and develop an ability to recall and apply knowledge rapidly.
- make use of diagrams and informal notes and jottings to help record steps and partial answers when using mental methods
- have an efficient, reliable, compact written method of calculation for each operation, which they can apply with confidence when undertaking calculations
- be able to identify when a calculator is the best tool for the task and use this primarily as a way of checking rather than simply a way of calculating.
- be able to explain their strategies to calculate and, using spoken language, give mathematical justification, argument or proof.

| The new bits |  |
| :--- | :--- |
| Reception | Children will count numbers to 20. <br> Children will double, halve and share numbers up to 20. |
| Year 1 | Children count to and across 100, forwards and backwards <br> beginning from any given number. <br> Children begin to use $\frac{1}{2}$ and $\frac{1}{4}$. |
| Year 2 | Children recognise, name and write the fractions 1/3, $\frac{1}{4}, 2 / 4$ and $\frac{3}{4}$ <br> of length, shapes and quantities. |
| Year 3 | Compare, order and calculate number totals up to 1000. <br> Begin to use columnar methods for addition and subtraction. <br> Count on and back in tenths. <br> Tell and write the time from an analogue clock and 12 and 24 hour <br> clocks. <br> Recognise Roman numerals from I to XII. (1 to 12) |
| Year 4 | Compare, order and calculate number totals up to $10,000$. <br> Multiply two and three-digit numbers by a one-digit number using <br> formal written method. <br> Recognise Roman numerals from I to $C$ (1 to 100) <br> Tell and write the time with accuracy using $24 h ~ n o t a t i o n . ~$ <br> Recognise and write decimal equivalents to $\frac{1}{4}, \frac{1}{2}$ and $\frac{3}{4}$. |
| Year 5 | Compare, order, round and calculate number totals up to 1,000,000 <br> and determine the value of each digit. <br> Recognise and use square and cubed numbers and use the notation <br> for these: 2 <br> Recognise and write Roman numerals from I to M (1 to 1000) |
| Year 6 | Compare, order, round and calculate number totals up to <br> $10,000,000$ and determine the value of each digit. <br> Use long multiplication to multiply multi-digit numbers by a two- <br> digit number. <br> Use formal short division and interpret remainders according to <br> context. |

## Foundation Stage 1 - addition

## Curriculum 2014 Statutory Requirements

Pupils should be taught to:
Birth -to 11 months - notice changes in number of objects / images, sounds in groups of and upto 3
8-20 months - has some understanding that things exist even when out of sight
16-26 months - Begins to organise and categorise objects -sorting
22-36 months - knows that a group of things changes in quantity when something is added or taken away
30-50-separates a group of 3 or 4 objects in different ways beginning to recognise that the total is still the same
40-60 - finds the total number of items in two groups by counting all of them
In practical activities and discussions begins to use the vocabulary involved in addition and subtraction


I can count six frogs.
Teaching Points

Use number lines 0-10

Numbers in the environment inside and outside

Putting objects in tens frames and numicon


## Foundation Stage 2 - addition

## Curriculum 2014 Statutory Requirements

Pupils should be taught to:
Early Learning Goal - Children count reliably with numbers from one to 20, place them in order and say which number is one more or one less than a given number. Using quantities and objects, they add and subtract two single-digit numbers and count on or back to find the answer.
Count on from first group to add two groups of objects


## Year 1 - addition

## Curriculum 2014 Statutory Requirements

Pupils should be taught to:

- read, write and interpret mathematical statements involving addition (+) and equals (=) signs THIS MEANS THE SAME AS - relate this to balance number sentences and scales
- represent and use number bonds and related subtraction facts within 20
- add one-digit and two-digit numbers to 20, including zero
- solve one-step problems that involve addition, using concrete objects and pictorial representations, and missing number problems such as $9=\square+7$.

Using a marked number line with marked divisions to 20 to solve calculations such as:
$9+7=\square$ Demonstrate with frogs jumping along the line
Number tracks

| 9 | 10 | 11 | 12 | 13 | 14 | 15 | 16 | 17 | 18 |
| :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- |

##  <br> $01223456789101 \mid 121314151617181920$

Appropriateness of number: choices of number here remain within 20 and build towards crossing 10.

Progress - numbered line, divisions with numbers on.
Begin to introduce $\square=9+7$ to show the symbolism of balanced calculations and commutative number sentences.

Working up from number bonds to $5,6,7,10,20$.
Practical equipment to support this addition work: Coat hangers, bead strings, 100 squares, Concrete objects, magic beans, numicon, base ten materials

Use part + part = whole as variation

## Teaching Points

Numbers to 20

Counting forward/up in jumps on top of the number line when adding.

Model the checking process as this is built upon throughout the strategies and policy.

Snakes and ladders game good to support

Ensure that children are being taught to count the jumps.

Variation ideas
$7+2=$
$17+2=$
$7+12=$
$9+6=$

$10+6=$
$11+6=$
$13+6=$
$8+3=$
$10+3=$
$12+3=$

## Year 2 - addition

## Curriculum 2014 Statutory Requirements

Pupils should be taught to:

- solve problems with addition:
- using concrete objects and pictorial representations, including those involving numbers, quantities and measures
- applying their increasing knowledge of mental and written methods
- recall and use addition facts to 20 fluently, and derive and use related facts up to 100
- add numbers using concrete objects, pictorial representations, and mentally, including:
- a two-digit number and ones
- a two-digit number and tens
- two two-digit numbers
- adding three one-digit numbers
- show that addition of two numbers can be done in any order (commutative) and subtraction of one number from another cannot
- recognise and use the inverse relationship between addition and subtraction and use this to check calculations and solve missing number problems
Once understanding of plcae value is secure using practical resources eg) base ten, bead strings, numicon etc, children will move to 'petal method' for adding two, two digit numbers.


Partitioning and applying addition mentally of partitioned numbers:
$40+30=70$
$47+36=83$
$7+6=13$

Progressing to expanded written, column Start with no crossing of tens, then onto crossing tens at the end of the year.

Ensure that the calculation done at each stage is written beside so children are aware of process.

|  |  | method: |
| ---: | :--- | :--- |
|    <br> 4 7  <br> + 6  |  |  |
| 1 | 3 | $(7+6)$ |
| 7 | 0 | $(40+30)$ |
| 8 | 3 |  |

## Teaching Points

Counting forward in units then tens. When counting in units, suggesting 'number bonds' and related facts to make jumps.

Headings of columns for addition are labelled

Note how appropriateness of number ensures that these numbers do not require carrying at this stage.

Variation ideas
$8+2=10$
$80+20=100$
$800+200=1000$
? $=8+2$
$10=?+2$
$100=?+20$

## Year 3 - addition

## Curriculum 2014 Statutory Requirements

Pupils should be taught to:

- add numbers mentally, including:
- a three-digit number and ones
- a three-digit number and tens
- a three-digit number and hundreds
- add numbers with up to three digits, using formal written methods of columnar addition
- estimate the answer to a calculation and use inverse operations to check answers
- solve problems, including missing number problems, using number facts, place value, and more complex addition.



## Year 4 －addition

## Curriculum 2014 Statutory Requirements

Pupils should be taught to：
－add with up to 4 digits using the formal written methods of columnar addition where appropriate
－estimate and use inverse operations to check answers to a calculation
－solve addition two－step problems in contexts，deciding which operations and methods to use and why．

## Formal written strategy modelled with：

Th H T O labelled in columns．
One digit per square．
Calculate from ones（least significant figure）．

| Th | $H$ | T | 0 |
| :--- | :--- | :--- | :--- |
| 4 | 6 | 2 | 7 |
| 3 | 9 | 1 | 4 |
| 8 | 5 | 4 | 1 |
| 1 |  | 1 |  |



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Teaching Points
Building on strategy from Year 3 moving to using numbers which， when added，remain within the 10，000 boundary．

Ensure clarity when adding two，four digit numbers and move to adding up to three integers including three－digit add four－ digit．

Progressing to the use of formal，compact method（modelling alongside expanded method）．

Variation ideas

## Year 5 - addition

## Curriculum 2014 Statutory Requirements

Pupils should be taught to:

- add whole numbers with more than 4 digits, including using formal written methods (columnar addition)
- add numbers mentally with increasingly large numbers
- use rounding to check answers to calculations and determine, in the context of a problem, levels of accuracy
- solve addition multi-step problems in contexts, deciding which operations and methods to use and why.

Building on Y4 strategy and number choices moving to numbers, when added within 1 million.

| TTh | Th | $H$ | $T$ | $O$ |
| :--- | :--- | :--- | :--- | :--- |
| 4 | 3 | 2 | 0 | 1 |
| 2 | 2 | 1 | 2 | 4 |
| 3 | 1 | 3 | 2 | 1 |
| 9 | 6 | 6 | 4 | 6 |

Progressing to addition of numbers to two decimal places in contex $\dagger$ (such as money $£$ including $€$ and $\$$ as appropriate)
$£ 132.52+£ 213.83$


Note appropriateness of number above where there is only one 'carry' initially to ensure clarity and understanding of the layout and process.

Estimating answers:
Rounding this
calculation to
nearest ten:
$£ 130+£ 210=$
£340

## Teaching Points

Note appropriateness of numbers: initially, when dealing with larger numbers, not requiring 'carrying' to ensure clarity and understanding of application of strategy moving swiftly to numbers requiring carrying.

Model when writing the answer, and when writing numbers such as that shown, the use of commas:
96,646
Use of rounding to check the relevance of numbers in answer.

When calculating using numbers involving decimals, a clear step to success must be the writing in of the decimal point in the answer area first to help when carrying past this boundary. Variation ideas:
$37+19=56$
$47+19=66$
$57+19=76$
$5.7+1.9=7.6$

## Year 6 - addition

## Curriculum 2014 Statutory Requirements

Pupils should be taught to:

- solve addition multi-step problems in contexts, deciding which operations and methods to use and why

Building on Y 5 strategy and number choices moving to numbers, when added within 10 million.

Children secure strategies for addition when adding more than two numbers including numbers to three decimal places.

| 1 | 2 | 0 | 5 | 3 | 7 |
| ---: | ---: | ---: | ---: | ---: | ---: |
| 2 | 3 | 4 | 2 | 7 | 1 |
| 3 | 2 | 3 | 2 | 2 | 1 |
| 6 | 7 | 8 | 0 | 2 | 9 |
|  |  | 1 | 1 |  |  |

Calculating decimal numbers to three decimal places:

$$
\begin{array}{lllll}
0 & \bullet & 5 & 5 & 7 \\
1 & \bullet & 2 & 1 & 1 \\
0 & \bullet & 2 & 0 & 2 \\
\hline 1 & \bullet & 9 & 7_{x} & 0
\end{array}
$$

## Foundation Stage 1 - subtraction

## Curriculum 2014 Statutory Requirements

Pupils should be taught to:
Birth -to 11 months - notice changes in number of objects / images, sounds in groups of and upto 3
8-20 months - has some understanding that things exist even when out of sight
16-26 months - Begins to organise and categorise objects -sorting
22-36 months - knows that a group of things changes in quantity when something is added or taken away
30-50-separates a group of 3 or 4 objects in different ways beginning to recognise that the total is still the same
40-60 - Understands subtraction as taking away objects from a group and counting on how many are left.
In practical activities and discussions begins to use the vocabulary involved in addition and subtraction


Teaching Points

Use number lines 0-10
Numbers in the environment inside and outside

I can count six frogs.


## Foundation Stage 2 - subtraction

## Curriculum 2014 Statutory Requirements

Pupils should be taught to:
Early Learning Goal Children count reliably with numbers from one to 20 , place them in order and say which number is one more or one less than a given number. Using quantities and objects, they add and subtract two single-digit numbers and count on or back to find the answer.

Count back from large group as you take away say count back as you take away.
$6-2=4$


Teaching Points

Counting and reading numbers to 20
Doubling using objects and numbers Halving using objects Sharing using objects Adding and subtracting two single digit numbers referring to a number line

## Year 1 - subtraction

## Curriculum 2014 Statutory Requirements

Pupils should be taught to:

- read, write and interpret mathematical statements involving subtraction (-) and equals (=) signs
- represent and use number bonds and related subtraction facts within 20
- subtract one-digit and two-digit numbers to 20, including zero
- solve one-step problems that involve subtraction, using concrete objects and pictorial representations, and missing number problems such as $9=\square-7$.

Sam spent 7p. What was his change from 20p?
Children use concrete, practical resources moving to images and physically 'cross off' or remove to ensure a real understanding of 'taking away'.

Pupils begin to explore missing number problems involving - and = notation.

| $7-3=\square$ | $\square=7-3$ |
| :--- | :--- |
| $7-\square=4$ | $4=\square-3$ |
| $\square-3=4$ | $4=7-\square$ |
| $\square-\nabla=4$ | $4=\square-\nabla$ |

Solving a problem such as: 19-7=

Counting back from original number on a numberline:


End of year - using counting on to find the difference. $\begin{array}{lllllllllll}1 & 2 & 3 & 4 & 5 & 6 & 7 & 8 & 9 & 10 & 11 \\ 12\end{array}$


Teaching Points When counting the remaining amount, and when checking that the correct number have been taken away, model efficient counting in twos where necessary or arrayed numbers of ten for example.

Model the checking process as this is built upon throughout the strategies and policy.

When solving missing number problems, ensure that there is a variety of layout where there is a modelling of 'balancing calculations.

Counting back along the top of the number line. Variation ideas

| $9-5=$ | $9-7=$ |
| :--- | :--- |
| $8-5=$ | $10-7=$ |
| $7-5=$ | $11-7$ |

## Year 2 - subtraction

## Curriculum 2014 Statutory Requirements

Pupils should be taught to:

- solve problems with subtraction:
- using concrete objects and pictorial representations, including those involving numbers, quantities and measures
- applying their increasing knowledge of mental and written methods
- recall and use subtraction facts to 20 fluently, and derive and use related facts up to 100
- subtract numbers using concrete objects, pictorial representations, and mentally, including:
- a two-digit number and ones
- a two-digit number and tens
- two two-digit numbers
- subtracting three one-digit numbers
- show that addition of two numbers can be done in any order (commutative) and subtraction of one number from another cannot
- recognise and use the inverse relationship between addition and subtraction and use this to check calculations and solve missing number problems

Building on strategies from Y1: using a number line to 'take away' and 'find the difference' by counting under or on the line respectively.

Start initially with a calculation such as 39-7.

Moving to calculations such as: 42-5 (crossing tens)


Model when using the strategy above to find the difference to 'jump' to the next ten to help make jumps more straight forward.

## Teaching Points

This calculation does not cross into the previous tens boundary to ensure clarity on the strategy and ensures understanding through subtracting a 'units only' initially.

Move to modelling counting on top of the line to 'find the difference' or under to 'take away'.

Children use a number line without divisions.

Model breaking down the whole number through partitioning and also, using bonds of numbers such as 2 and $5=7$ as shown.

## Variation ideas

$37-6=$

## Updated - Autumn Term 2016



## Year 3 - subtraction

## Curriculum 2014 Statutory Requirements

Pupils should be taught to:

- subtract numbers mentally, including:
- a three-digit number and ones
- a three-digit number and tens
- a three-digit number and hundreds
- a three-digit number and thousands
- subtract numbers with up to three digits, using formal written methods of columnar subtraction
- estimate the answer to a calculation and use inverse operations to check answers
- solve problems, including missing number problems, using number facts, place value, and more complex subtraction.

Calculating subtractions from numbers up to 1000.

Model deciding appropriate calculation choices: calculations such as:
296-5 or 296-35 should be tackled mentally.
As pupils move towards formal, columnar written strategies, begin by modelling the value and layout practically


For example, model 346-123 using practical resources.


Move to formal columnar strategy using

$\qquad$
ㅁㅁ numbers not requiring exchange before strategy and understanding is secure.

No Exchange at this stage!

| $H$ | $T$ | $O$ |  |
| :--- | :--- | :--- | :--- |
| $-\quad 2$ | 9 | 8 |  |
| 1 | 7 | 9 |  |
|  |  | 9 | $(18-9)$ |
|  | 1 | 0 | $(80-70)$ |
| 1 | 0 | 0 | $(200-100)$ |

## Year 4 - subtraction

## Curriculum 2014 Statutory Requirements

Pupils should be taught to:

- subtract with up to 4 digits using the formal written methods of columnar subtraction where appropriate
- estimate and use inverse operations to check answers to a calculation
- solve subtraction two-step problems in contexts, deciding which operations and methods to use and why.
Pupils calculate subtractions from numbers up to 10,000 and beginning to explore decimals in the context of currency ( $£$ ).

Pupils use columnar written strategies to calculate building upon that from Year 3. As with Year 3, model layout and move to subtraction with the need for exchange using practical materials initially and progressing from 3-digit subtracting a 3-digit to 4digit subtracting 3 and 4-digit integers. Take the Year 3 demonstration as the starting point in exchange.


Progressively move towards 4-digit subtract 3-and 4-digit where again, only one exchange is needed initially.

## Teaching Points

Note that when modelling practically, and until secure, only one exchange per calculation is required.

Note at the point of physical exchange that the value of the number remains the same (there is still 346, some are simply exchanged).

Modelling of formal written must, initially, occur alongside the practical examples.

When moving to formal columnar method, ensure a progressive learning sequence where only one exchange is required and move this along when secure.

When modelling formal

| Progressing to subtraction of numbers to two decimal places in context (such as money $£$ including $€$ and $\$$ as appropriate) | written calculations, model placing a decimal point in the 'answer line' before commencing subtracting from the least significant figure. |
| :---: | :---: |
| $£ 213.83-£ 183.51$ |  |
| $H$ $T$ $U \bullet t$ $h$ <br> ${ }^{1} 2$ 1 $3 \bullet 8$ 3 <br> 1 8 $3 \bullet 5$ 1 Estimating answers: <br> Rounding this calculation to nearest <br> ten: $£ 210-£ 180=£ 30$ |  |
| $0 \quad 3003$ |  |

## Year 5 - subtraction

## Curriculum 2014 Statutory Requirements

Pupils should be taught to:

- subtract whole numbers with more than 4 digits, including using formal written methods (columnar subtraction)
- subtract numbers mentally with increasingly large numbers
- use rounding to check answers to calculations and determine, in the context of a problem, levels of accuracy
- solve subtraction multi-step problems in contexts, deciding which operations and methods to use and why.
Strategies build on those of Year 4 and involve starting numbers of up to 100,000 and progressing to 1,000,000.


## Formal Written:

Progressively, and before moving to larger numbers, begin to explore written strategies where ' 2 exchanges' are needed:


Estimating answers:
E: 7900-2600 =

Progressively move to calculations such as:
14,067-11,850 =

## Mental Strategies:

When modelling and teaching mental strategies, refer to picturing the use of a number line and either counting back or on: $\Delta=12,462-2,300$

10,162
$12,162 \leftarrow$
$\begin{array}{lll}-2,000 & -300 & 12,462\end{array}$

Teaching Points Discrete teaching of the notion of more than one exchange must be taught discretely, and does exchanging through a 0 as shown. Modelling here how an exchange is needed and is placed alongside a prior exchange.

Modelling and checking against estimates is a key part of the calculation process to ensure an understanding and automatic check of validity. Note use of, to separate chunks of numbers in 'number sentences' but not in columnar strategy. Note use of symbols and algebraic symbols such as $x$ or $y$ to find missing values.

When modelling mental methods, promote values in red as being jottings.

## Year 6 - subtraction

## Curriculum 2014 Statutory Requirements

Pupils should be taught to:

- solve subtraction multi-step problems in contexts, deciding which operations and methods to use and why
Strategies build on those of Year 5 and involve starting numbers of up to 1,000,000 and progressing to 10,000,000.

Pupils apply their learning of subtraction strategies and combine these with other areas of learning to solve problems such as:

$$
\begin{aligned}
& 632,465+(745,676-325,534)= \\
& \text { progressing to } \\
& 8,675,509-(9,645,253-2,867,675)=
\end{aligned}
$$

Pupils apply written subtraction skills to numbers up to and including three decimal places (3dp). These are presented in contextual situations such as units of measure.

Calculations and ranges of numbers are applied through worded problems including units of measure.

Calculations to include examples such as:

## Teaching Points

Model the use of brackets in multi-step problems identifying brackets as the initial step needed and combining this with an additional written strategy.

Refer at these stages, as taught in previous years to estimation recorded as $\mathrm{E}=$.

Here, discrete and modelled teaching of 'selecting the appropriate strategy' must be taught.

For this example, counting on mentally, or with jottings referring back to knowledge of number lines would work best.

Here, a formal, columnar subtraction strategy will be more effective.

## Foundation Stage 2 - multiplication

## Curriculum 2014 Statutory Requirements

Pupils should be taught to:
Early Learning Goal Children count reliably with numbers from one to 20, place them in order and say which number is one more or one less than a given number. Using quantities and objects, they add and subtract two single-digit numbers and count on or back to find the answer. They solve problems, including doubling, halving and sharing.

Doubling machines, ladybirds with spots, symmetry, dice, making it clear that doubling is adding the same number. Doubling songs

$6+6=12$ - Making it clear you are adding the same number.

Teaching Points

Doubling using objects and numbers

## Year 1 - multiplication

## Curriculum 2014 Statutory Requirements

Pupils should be taught to:

- solve one-step problems involving multiplication, by calculating the answer using concrete objects, pictorial representations and arrays with the support of the teacher.
Pupils build on learning in the Foundation Stage and ensure a clear understanding of the concept of doubling before moving. Using concrete objects, image representations and the use of physical or images of arrays, pupils solve problems such as:

There are three sweets in one bag. How many sweets are in five bags?


There are three fish in one tank. How many fish are in four tanks?


Use different objects to add equal groups.

## Teaching Points

Note that when using worded problems, the language aspect of this must be accessible here, the use of talking tins or image based questioning might be needed to ensure equality of access to the mathematics aspect of the question.

Key vocabulary - Lots of
Make links with repeated addition

Ensure that pupils experience contextual links such as:


Counting in steps of 2,5 and 10 - tables progression

## Year 2 - multiplication

## Curriculum 2014 Statutory Requirements

Pupils should be taught to:

- recall and use multiplication facts for the 2,5 and 10 multiplication tables, including recognising odd and even numbers
- calculate mathematical statements for multiplication within the multiplication tables and write them using the multiplication ( $x$ ) and equals ( $=$ ) signs
- show that multiplication of two numbers can be done in any order (commutative) and division of one number by another cannot
- solve problems involving multiplication and division, using materials, arrays, repeated addition, mental methods, and multiplication and division facts, including problems in contexts.

Pupils recall and use $2 x 5 x 10 x$ and $3 x$ table but use doubling to progress onto $4 x$

## Start initially with recap of arrays:

Create arrays using counters/ cubes to show multiplication sentences.


## Build on repeated addition

$3 \times 4=12$
Also demonstrate this is the same as $4 \times 3$


Pupils explore, practically, commutative multiplication facts showing that the same product is produced.

Teaching Points Here, build upon partitioning skills to partition and then multiply to strengthen links between place value and partitioning.

Model practically with place value arrow cards to model multiplication steps.

When introducing grid method, referring to it as such, model initially alongside partitioning strategy.

Note appropriateness of number where numbers remain initially in 'teens' to strengthen ability to multiply a digit by 10 .

Variation ideas
$2 \times 3=$
$2 \times 30=$
$2 \times 300=$
$20 \times 3=$
$200 \times 3=$

## Year 3 - multiplication

## Curriculum 2014 Statutory Requirements

Pupils should be taught to:

- recall and use multiplication facts for the 3,4 and 8 multiplication tables
- write and calculate mathematical statements for multiplication using the multiplication tables that they know, including for two-digit numbers times one-digit numbers, using mental and progressing to written methods
- solve problems involving missing number problems involving multiplication including positive number scaling problems and correspondence problems where n objects are connected to m objects.


## Pupils recall and use $2 x 5 \times 10 \times 3 \times 4 x 6 x 8 x$ and 9x

Tables knowledge builds on using doubling skills of $2 x$ to find $4 x$ and then doubling $4 x$ to find $8 x$ emphasising efficiency and using known facts.
Pupils solve problems such as $34 \times 3$ using the grid method.
Model calculating this alongside the expanded preparing for short multiplication in Year 4 - teach alongside.


## Teaching Points

Use base 10 to support

Note how digits in numbers are, initially, those that are being reinforced and taught through expected multiplication tables knowledge.

Variation ideas
$9 \times 8=$
$9 \times 80=$
$9 \times 800=$
$90 \times 8=$
$900 \times 8=$
? $=900 \times 8$
$72=? \times 8$

## Year 4 - multiplication

## Curriculum 2014 Statutory Requirements

Pupils should be taught to:

- recall and use multiplication facts for multiplication tables up to $12 \times 12$
- use place value, known and derived facts to multiply mentally, including: $x 0 \times 1$ and multiplying together three numbers
- recognise and use factor pairs and commutativity in mental calculations
- multiply two-digit and three-digit numbers by a one-digit number using formal written layout
- solve problems involving multiplying, including the distributive law to multiply two-digit numbers by one-digit including positive number scaling problems and correspondence problems where n objects are connected to m objects.

Pupils recall and use tables facts up to $12 \times 12$

Building on the strategies from Year 3, pupils move towards multiples of ten based on the known table facts from above such as $30 x$ and 40x.

Calculations are completed progressing from 2-digit $\times 1$ digit to 3-digit (1[][] $\times[]$ ) $\times$ 1-digit.

Calculations continues an 'expanded' formal written methods:

| $H$ | $T$ | $O$ |  |
| :---: | :---: | :---: | :---: |
| 1 | 4 | 3 |  |
|  | $X$ | 6 |  |
|  | 1 | 8 | $(3 \times 6)$ |
| 2 | 4 | 0 | $(40 \times 6)$ |
| 6 | 0 | 0 | $(100 \times 6)$ |
| 8 | 5 | 8 |  |

Teaching Points
Note here that number choice ensures that columnar addition is supported in this example where 'carrying' of numbers is not required for the strategy to work.

Model brackets to show calculation to ensure and check understanding

Where columnar addition is secure, progress to applying carrying here. Pupils reinforce $\times 10$ and $\times 100$ through conversions of units of measure in contextual situations.

Variation ideas
$6 \times 7=$
$6 \times 70=$
$6 \times 700=$
$60 \times 7=$
? $=600 \times 7$
$0.6 \times 7=$
$0.6 \times 70=$

## Year 5 - multiplication

## Curriculum 2014 Statutory Requirements

Pupils should be taught to:

- identify multiples and factors: all factor pairs of a number, common factors of two numbers, establish whether a number up to 100 is prime and recall prime numbers up to 19
- multiply numbers up to four digits by a one- or two-digit number using a formal written method - multiply whole numbers and those involving decimals by 10, 100 and 1000.

| Autumn Term - Introduce short multiplica |  |  |
| :--- | :---: | :--- |
| H | T | 0 |
| 1 | 4 | 3 |
| $x$ |  | 6 |
| 8 | 5 | 8 |
| 2 | 1 |  |

Progress on to $2 \times 2$ in Spring Term

Using an expanded, columnar multiplication strategy, pupils multiply numbers such as:
$37 \times 29$

|  |  | T | 0 |  |
| :---: | :---: | :---: | :---: | :---: |
|  |  | 3 | 7 |  |
|  | $x$ | 2 | 9 |  |
|  |  | 6 | 3 | (9×7) |
|  | 2 | 7 | 0 | (9x30) |
|  | 1 | 4 | 0 | (20×7) |
|  | 6 | 0 | 0 | ( $20 \times 30$ ) |
| 1 | 0 | 7 | 3 |  |
|  | X |  |  |  |

Teaching Points Note here that this strategy and number choices rely on an ability to use columnar addition efficiently and accurately. Those pupils needing support here can revert to grid but progress to expanded formal as soon as is practicably possible.

Note modelling of noting steps to help with selfchecking and ensuring knowledge of place value.

Multiply by ones, explain when multiplying by tens, the numbers will be 10 times bigger, digits move to the left on one place as a result. 0 is a place holder.

## Year 6 - multiplication

## Curriculum 2014 Statutory Requirements

Pupils should be taught to:

- identify multi-digit numbers up to 4 digits by a two-digit number using formal, long multiplication
- identify common factors, common multiples and common prime numbers
- use their knowledge of the order of operations to carry out calculations involving the four operations


Progress to three-digit $\times 2$-digit and TU.t $\times U$ using expanded formal. Move to Year 6 strategy where these numbers are confident.

|  |  | T | 0 |  | $1 / 10$ |
| :---: | :---: | :---: | :---: | :---: | :---: |
|  | 3 | 6 | $\bullet$ | 2 |  |
| $\times$ |  | 7 |  |  |  |
|  |  | 1 |  |  | 4 |
|  | 4 | 2 | $\bullet$ | 0 |  |
| 2 | 1 | 0 |  |  | 0 |
| 2 | 5 | 3 |  | 4 |  |

## Teaching Points

Build here from 'teens' to 20s and reinforce efficiency where this number could apply $\times 10$ and doubling knowledge.

Variation
Be aware of how calculation maybe in different order. Progress onto missing numbers in the calculation.

4 \begin{tabular}{ccccc}
2 \& 3 \& 1 \& 4 <br>
$\times$ \& \& 2 \& 3 <br>
\hline

 

<br>
\hline \& 2 \& 8 \& 0 <br>
6 \& 9 \& 4 \& 2
\end{tabular}

$5 x_{x} 3_{x} 2$

## Foundation Stage 2 - division

## Curriculum 2014 Statutory Requirements

Pupils should be taught to:
Early Learning Goal Children count reliably with numbers from one to 20, place them in order and say which number is one more or one less than a given number. Using quantities and objects, they add and subtract two single-digit numbers and count on or back to find the answer. They solve problems, including doubling, halving and sharing.

Practically halving objects - both halves being exactly the same size - Start with play dough and things you can cut and then progress to practical objects.
Eg: Pizza - cut in half to make two pieces - then add toppings, eg: 2 tomatoes - half of 2 is 1 .
Making number stories is a focus for understanding


## Teaching Points

Counting and reading numbers to 20

Doubling using objects and numbers

Halving using objects

Sharing using objects

Food as a focus for understanding

## Year 1 - division

## Curriculum 2014 Statutory Requirements

Pupils should be taught to:

- solve one-step problems involving division, by calculating the answer using concrete objects, pictorial representations and arrays with the support of the teacher.
Pupils begin by reinforcing prior learning where division is understood by grouping and sharing:

Develop halving understanding of shapes and numbers initially and move on to quarters.

12 girls play a game in groups of 4 . How many are in each group?


Pupils begin to explore related division facts and linking these directly to inverse, commutative facts:
$6 \div 2=$
$\square=6 \div 2$
$6 \div \square=3$
$3=6 \div \square$
$\square \div 2=3$
$3=\square \div 2$
$\square \div \nabla=3$
$3=\square \div \nabla$

Sharing of 'chunks' begins to be modelled physically on a number line:
$8 \div 2=$ "How many $2 s$ make 8?"
$\begin{array}{lllllllllllllllllll}0 & 1 & 2 & 3 & 4 & 5 & 6 & 7 & 8 & 9 & 10 & 11 & 12 & 13 & 14 & 15 & 16 & 17 & 18 \\ 19 & 20\end{array}$

## Year 2 - division

## Curriculum 2014 Statutory Requirements

Pupils should be taught to:

- recall and use multiplication and division facts for the 2,3,5 and 10 multiplication tables, including recognising odd and even numbers
- calculate mathematical statements for division within the multiplication tables and write them using the signs $\div$ and $=$
- show that multiplication of two numbers is commutative but division is not
- solve problems involving division using materials, arrays, repeated addition, mental methods and division facts, including problems in contexts.
Calculations here build on expected known multiplication facts where division is by a divisor or 2,5 and 10 initially progressing to Y 3 multiplication facts of 3 .


Link division
to
multiplication
by creating
an array and
thinking
about the
number sentences that can be created.
Eg $15 \div 3=5 \quad 5 \times 3=15$
$15 \div 5=3 \quad 3 \times 5=15$
$18 \div 3$ can be modelled as sharing - 18 shared between 3 or modelling jumping back in threes to share in 'chunks' of 3:


Pupils continue to explore division as sharing and grouping:

Teaching Points

Variation
$2 \times 3=6$
$3 \times 2=6$
$6 \div 3=2$
$6 \div 2=3$

Model counting jumps 'chunks' on number line.

Move on to grouping - How many 3's make 18?


## Year 3 - division

## Curriculum 2014 Statutory Requirements

Pupils should be taught to:

- recall and use multiplication and division facts for the 3,4 and $8 \times$ tables
- write and calculate mathematical statements for division using the multiplication tables they know, including 2-digit divided by 1-digit using mental and progressing to formal written methods
- solve problems, involving missing number problems, involving division, including positive number scaling problems and correspondence problems where $n$ objects are connected to $m$ objects.

Informal Written Method (Bus Stop Method)
$36 \div 4=$


Write out $4 \times$ tables:
$4,8,12,16,20,24$, etc

Using the chunking method, pupils begin to divide 2-digit numbers by multiplication facts (one-digit) that are expected to be fluent at this stage progressing to any single digit divisor.

Progress on to division with remainders:
$53 \div 4=$
4
Children need to be reminded to always check their answers with the inverse

Teaching Points

Teacher models the layout of a calculation where there are the following key features:

First ten tables facts to build on recall and also, to promote a habit to be referred to later on in the progressive division strategies.

Chunks noted in brackets to count up (not the divisor (4) as this can lead to adding this as a chunk).

First key question as a step to success is 'Can I take a chunk of 10x?'

Appropriateness of number: these numbers do not need an exchange in the subtraction element of the strategy.

## Year 4 - division

## Curriculum 2014 Statutory Requirements

Pupils should be taught to:

- recall multiplication and division facts up to $12 \times 12$
- use place value, known and derived facts to divide mentally, including dividing by 1
- solve problems involving dividing a three-digit number by one-digit and number using a formal layout
Ensuring an understanding of the relationship between $\div$ and $X$, pupils build on chunking from Year 3 to use this strategy to divide 3-digit numbers by 1- and 2-digit numbers:
$432 \div 5=$



## Teaching Points (Non statutory) Build here from numbers without a remainder using this strategy progressing to a single digit remainder.

Chunks noted in brackets to count up (not the divisor (4) as this can lead to adding this as a chunk).

First key question as a step to success is 'Can I take a chunk of 10x, 100x or a multiple of 10x?' (This will be modelled by teacher by applying using known facts and place value.
Greater depth - Here, remainders can begin to be expressed as a fraction. Here, appropriateness of number enables this to be expressed as a decimal with ease. $2 / 5=0.4$

## Year 5 - division

## Curriculum 2014 Statutory Requirements

Pupils should be taught to:

- identify multiples and factors, including finding all factor pairs of a number, common factors of two numbers, know and use the vocabulary of prime numbers and establish whether a number up to 100 is prime
- multiply and divide numbers mentally drawing on known facts
- divide numbers up to 4 digits by a one-digit number using a written method and interpret remainders appropriately for the context
- divide whole numbers and those involving decimals by 10,100 and 1000.

Pupils develop use of the short division method started in Year
4.

Pupils apply short division strategy to solve questions such as: $1176 \div 6=$
$\begin{array}{r}196 \\ \hline 115736\end{array}$

Pupils apply this with a remainder
$6{ }^{196759} r 3$

Then interpret the remainder as a fraction $3 / 6$ then a decimal - 0.5
196.5
6115739.30

## Year 6 - division

## Curriculum 2014 Statutory Requirements

Pupils should be taught to:

- divide numbers up to 4 digits by a two-digit number using the formal written method of long division, and interpret remainders as whole number remainders, fractions, or by rounding as appropriate for the context.
- divide numbers up to 4 digits by a two-digit number using the formal written method of short division as appropriate.

Pupils use long division to calculate:
$432 \div 15=$

This answer can be shown as a quotient (rather than an integer remainder): 28 12/15 = $284 / 5$

Progressing to long division to find a decimal remainder:


Simplify this method into long division

$$
28.8
$$

15
$4 \quad 3^{13} \mathbf{2 l}^{120}$

## Year 1 - Fractions

Pupils should be taught to:

- Recognise, find and name a half as one of two equal parts of an object, shape or quantity.
- Recognise, find and name a quarter as one of four equal parts of an object, shape or quantity.


## Year 2 - Fractions

Pupils should be taught to:

- Recognise, find, name and write fractions $\frac{1}{3}, \frac{1}{4}, \frac{2}{4}$ and $\frac{3}{4}$ of a length, shape, set of objects or quantity -Write simple fractions for example, $\frac{1}{2}$ of $6=3$ and recognise the equivalence of $\frac{2}{4}$ and $\frac{1}{2}$.


## Year 3 - Fractions

Pupils should be taught to:

- Count up and down in tenths; recognise that tenths arise from dividing an object into 10 equal parts and in dividing one-digit numbers or quantities by 10
- Recognise, find and write fractions of a discrete set of objects: unit fractions and non-unit fractions with small denominators
- Recognise and use fractions as numbers: unit fractions and non-unit fractions with small denominators
- Recognise and show, using diagrams, equivalent fractions with small denominators

Add and subtract fractions with the same denominator within one whole:
Eg: 8/12 $+3 / 12=11 / 12$ Teaching point-add
numerator - ensure children recognise what a whole looks like.


Compare and order unit fractions, and fractions with the same denominators

## Year 4 - Fractions

## Pupils should be taught to:

- Recognise and show, using diagrams, families of common equivalent fractions
- Count up and down in hundredths; recognise that hundredths arise when dividing an object by one hundred and dividing tenths by ten.

Add and subtract fractions with the same denominator
$3 / 8+5 / 8=8 / 8$ same as 1 whole
$6 / 7-4 / 7=2 / 7$ Teaching point is subtracting the numerator

## Year 5 - Fractions

Pupils should be taught to:

- Compare and order fractions whose denominators are all multiples of the same number
- Identify, name and write equivalent fractions of a given fraction, represented visually, including tenths and hundredths
- Add and subtract fractions with the same denominator and denominators that are multiples of the same number

Add and subtract fractions with the same denominator and denominators that are multiples of the same number

Recognise mixed numbers and improper fractions and convert from one form to the other and write mathematical statements as a mixed number

For example, $\frac{2}{5}+\frac{4}{5}=\frac{6}{5}=1 \frac{1}{5}$
$1 / 8+1 / 8=2 / 8=1 / 4$
$\frac{1}{4}+1 / 8=3 / 8-\frac{1}{4}=2 / 8+1 / 8=3 / 8$
Multiply proper fractions and mixed numbers by whole numbers, supported by materials and diagrams
$1 / 5 \times 3=3 / 5$
$2 / 5 \times 4=8 / 5$

## Year 6 - Fractions

Pupils should be taught to:

- Use common factors to simplify fractions; use common multiples to express fractions in the same denomination
- Compare and order fractions, including fractions >1


## Add and subtract fractions with different denominators and mixed numbers, using the concept of equivalent fractions

Important: you multiply both top and bottom by the same amount, to keep the value of the fraction the same

Now the fractions have the same bottom number ("6"), and our question looks like this:


The bottom numbers are now the same, so we can go to step 2 .

Step 2: Add the top numbers and put them over the same denominator:

$$
\frac{2}{6}+\frac{1}{6}=\frac{2+1}{6}
$$

Here you can see it with pizza ...

## Multiply simple pairs of proper

 fractions, writing the answer in

In picture form it looks like this:

its simplest form for example, $1 / 2 \times 2 / 5$

Example:

$$
\frac{1}{2} \times \frac{2}{5}
$$

Step 1. Multiply the top numbers:

$$
\frac{1}{2} \times \frac{2}{5}=\frac{1 \times 2}{}=2
$$

Step 2. Multiply the bottom numbers:

$$
\frac{1}{2} \times \frac{2}{5}=\frac{1 \times 2}{2 \times 5}=\frac{2}{10}
$$

Step 3. Simplify the fraction:

$$
\frac{2}{10}=\frac{1}{5}
$$

Divide proper fractions by whole numbers for example, $\frac{1}{3} \div 2=\frac{1}{6}$
$\frac{1}{2}$ divided by $3=$

$2 \times 3$ 6

When half a pizza is divided into 3 equal parts. Each person gets one sixth of a whole pizza.


Divided
by 3:

## Foundation - key vocabulary

Adding and subtracting
add, more, and
make, sum, total
altogether
score
double
one more, two more, ten
more...
how many more to make... ?
how many more is... than...?
take (away), leave
how many are left/left over?
how many have gone?
one less, two less... ten less...
how many fewer is... than...?
difference between
is the same as

Solving problems
Reasoning about numbers or shapes
pattern
puzzle
answer
right, wrong
what could we try next?
how did you work it out?
count, sort
group, set
match
same, different
list

Problems involving 'real life' or money
compare
double
half, halve
pair
count out, share out
left, left over
money
coin
penny, pence, pound
price
cost
buy
sell
spend, spent
pay
change
dear, costs more
cheap, costs less, cheaper
costs the same as
how much...? how many...?
total

## Year 1 - key vocabulary

Words new to Year 1 are in red
Addition and subtraction +, add, more, plus make, sum, total
altogether
score
double, near double
one more, two more... ten more
how many more to make...?
how many more is... than...?
how much more is...?
-, subtract, take (away), minus leave

Multiplication and division lots of, groups of
$x$, times, multiply, multiplied by
once, twice, three times, four times, five times... ten times...
times as (big, long, wide and so on)
repeated addition
array
row, column
double, halve
share, share equally
one each, two each, three

Solving problems
Making decisions and reasoning
pattern
puzzle
answer
right, wrong
what could we try next?
how did you work it out?
count out, share out, left,
left over
number sentence
sign, operation

## Updated - Autumn Term 2016

| how many are left/left over? | each... |  |
| :--- | :--- | :--- |
| how many are gone? | group in pairs, threes... tens |  |
| one less, two less, ten less... | equal groups of |  |
| how many fewer is... than...? | , divide, divided by, divided <br> how much less is...? <br> difference between |  |
| half, halve left, left over |  |  |
| $=$, equals, sign, is the same as |  |  |

## Year 2 - key vocabulary

Words new to Year 2 are in red
Addition and subtraction
+, add, addition, more, plus make, sum, total
altogether
score
double, near double
one more, two more... ten more... one hundred more how many more to make...? how many more is... than...?
how much more is...?
-, subtract, take away, minus leave, how many are left/left over?
one less, two less... ten less...
one hundred less
how many less is... than...?
how much fewer is...?
difference between
half, halve
=, equals, sign, is the same as tens boundary

## Year 3 - key vocabulary

Words new to Year 3 are in red
Addition and subtraction
+, add, addition, more, plus
make, sum, total
altogether
score
double, near double one more, two more... ten more... one hundred
more
how many more to make ...?
how many more is... than ...?
how much more is...?

Multiplication and division lots of, groups of
$x$, times, multiply, multiplied by
multiple of
once, twice, three times, four times, five times... ten times...
times as (big, long, wide and so on)
repeated addition array
row, column
double, halve
share, share equally one each, two each, three each...
group in pairs, threes... tens equal groups of
$\div$ divide, divided by, divided into, left, left over

Multiplication and division lots of, groups of $x$, times, multiplication, multiply, multiplied by multiple of, product once, twice, three times, four times, five times... ten times...
times as (big, long, wide and so on)
repeated addition
array
row, column
double, halve

Solving problems
Making decisions and reasoning
pattern, puzzle calculate, calculation mental calculation jotting
answer
right, correct, wrong what could we try next? how did you work it out? number sentence sign, operation, symbol

## Solving problems

Making decisions and reasoning
pattern, puzzle calculate, calculation mental calculation
method
jotting
answer
right, correct, wrong what could we try next? how did you work it out?

| -, subtract, take (away), <br> minus <br> leave, how many are left/left over? <br> one less, two less... ten less... one hundred less <br> how many fewer is... than ...? <br> how much less is...? <br> difference between <br> half, halve <br> $=$, equals, sign, is the same as tens boundary, hundreds boundary | share, share equally one each, two each, three each... <br> group in pairs, threes... tens equal groups of <br> $\div$ divide, division, divided by, divided into <br> left, left over, remainder | number sentence sign, operation, symbol, equation |
| :---: | :---: | :---: |
| Year 4 - key vocabulary |  |  |
| Words new to Year 4 are in red <br> Addition and subtraction add, addition, more, plus, increase sum, total, altogether score double, near double how many more to make...? subtract, subtraction, take away, minus, decrease leave, how many are left/left over? <br> difference between half, halve how many more/fewer is... than...? <br> how much more/less is...? is the same as, equals, sign tens boundary, hundreds boundary inverse | Multiplication and division lots of, groups of times, multiplication, multiply, multiplied by multiple of, product once, twice, three times four times, five times... ten times <br> times as (big, long, wide, and so on) repeated addition array row, column double, halve share, share equally one each, two each, three each... <br> group in pairs, threes... tens equal groups of divide, division, divided by, divided into, divisible by remainder factor, quotient inverse | Solving problems Making decisions and reasoning pattern, puzzle calculate, calculation mental calculation method jotting answer right, correct, wrong what could we try next? how did you work it out? number sentence sign, operation, symbol, equation |
| Year 5 - key vocabulary |  |  |
| Words new to Year 5 are in | Multiplication and division | Solving problems |


| red <br> Addition and subtraction add, addition, more, plus, increase sum, total, altogether score double, near double how many more to make...? subtract, subtraction, take (away), minus, decrease leave, how many are left/left over? <br> difference between half, halve how many more/ fewer is... than...? <br> how much more/less is...? equals, sign, is the same as tens boundary, hundreds boundary units boundary, tenths boundary inverse | lots of, groups of times, multiply, multiplication, multiplied by multiple of, product once, twice, three times four times, five times... ten times <br> times as (big, long, wide, and so on) <br> repeated addition array <br> row, column <br> double, halve <br> share, share equally <br> one each, two each, three each... <br> group in pairs, threes... tens equal groups of divide, divided by, divided into, divisible by, divisor remainder factor, quotient, divisible by inverse <br> long division / multiplication short division / multiplication | Making decisions and reasoning pattern, puzzle calculate, calculation mental calculation method, strategy jotting answer right, correct, wrong what could we try next? how did you work it out? number sentence sign, operation, symbol, equation |
| :---: | :---: | :---: |
| Year 6 - key vocabulary |  |  |
| Words new to Year 6 are in red <br> Addition and subtraction add, addition, more, plus, increase sum, total, altogether score double, near double how many more to make...? subtract, subtraction, take (away), minus, decrease leave, how many are left/left over? <br> difference between | Multiplication and division lots of, groups of times, multiplication, multiply, multiplied by multiple of, product once, twice, three times four times, five times... ten times times as (big, long, wide, and so on) repeated addition array, row, column double, halve share, share equally | Solving problems Making decisions and reasoning <br> pattern, puzzle calculate, calculation mental calculation method, strategy jotting answer <br> right, correct, wrong what could we try next? how did you work it out? number sentence sign, operation, symbol, |


| half, halve <br> how many more/fewer is... <br> than...? <br> how much more/less is...? <br> is the same as, equals, sign <br> tens boundary, hundreds <br> boundary <br> units boundary, tenths <br> boundary <br> inverse <br> amount <br> brackets <br> calculator: clear, display, <br> enter, <br> key, memory, <br> ehange (money) <br> group in pairs, threes... tens <br> equal groups of <br> divide, division, divided by, <br> divided into <br> remainder <br> factor, quotient, divisible by <br> inverse <br> divisible by, divisor <br> remainder <br> lommutative | equation |  |
| :--- | :--- | :--- |
| lomplements (in 10, 100) | short division / multiplication |  |
| comrency |  |  |
| discount |  |  |
| exact, exactly |  |  |
| exchange rate |  |  |
| most/least significant digit |  |  |$\quad$|  |
| :--- |

